Girls Power Math 2019: Materials and Solutions

Dear Participant,

Thank you for attending Girls Power Math 2019! It was an inspiring experience to work with over a hundred young mathematicians for an afternoon of problem solving and collaboration. We're excited to continue providing socially empowering environments for middle school girls to tackle challenging STEM problems together.

The stations in this packet were designed and written by high school students at the Washington Student Math Association for our event hosted on January 26th, 2019. Each station is designed to be a challenge, and participants were encouraged to work together to find creative solutions - many of which may require students to learn new methods outside of their comfort zone. For us, it was a cause for celebration that the majority of the participants at the event were able to complete three stations, earning them an award.

As you review these materials, please keep in mind that by design they are meant to be challenging for the typical middle school student. If you are a student yourself, we hope that these stations can provide a helpful learning opportunity. For parents and educators, we hope these stations can be useful when reproduced at home or in the classroom.

Please feel free to reach out to us at contact@wastudentmath.org with any questions.

Best,
The WSMA Team

Station 1 (Probability): Participants will be asked to count, measure uncertainty and (as a challenge) calculate expected value and payoffs.
Author: Pravir Chugh

Station 2 (Chess): Participants arrange and use chess pieces for problem solving.
Authors: Laasya Nagareddy, Shifa Somji, Luke Xie

Station 3 (Origami): Participants will learn to fold a hexaflexagon - a mathematically unique origami form.
Authors: Vivian Lu, Laasya Nagareddy

Station 4 (Sudoku): Participants will play sudoku, testing speed and accuracy.
Author: Christina Yao

Station 5 (Tangrams): Participants will make tangrams: a dissection puzzle consisting of seven flat shapes, called tans, which are put together to form shapes.
Authors: Vivian Lu, Christina Yao, Anish Nagareddy

Station 6 (24): Participants will use mathematical operations to get the number 24 from the numbers drawn from 4 cards.
Authors: Laasya Nagareddy, Vivian Lu

Station 7 (Bingo): Participants will play bingo, but with a mathematical twist.
Authors: Laasya Nagareddy, Anish Nagareddy, Vivian Lu

Station 8 (Codebusters): Participants will learn about two ciphers, the Caesar Cipher and the Affine Cipher, and use these to encode and decode problems.
Authors: Laasya Nagareddy, Simon Kwon

Station 9 (Triangles): Participants will be given a paper triangle and will use a neat mathematical trick to cut the triangle into a perfect square.
Authors: Christina Yao, Laasya Nagareddy, Vivian Lu

Station 10 (Dimensional Analysis): Participants will apply estimation and dimensional analysis to tackle real world questions involving conversion factors.
Author: Pravir Chugh

Station 1: Probability (Participants)

Author: Pravir Chugh

Probability is a mathematical tool that mathematicians, scientists, and many more use to measure chance and uncertainty. It has applications in many real-world areas, such as understanding medicine, predicting weather, statistics, and analyzing sports.

Probability can be understood as:

$$\text{Probability of an event happening} = \frac{\text{Number of ways it can happen}}{\text{Total number of outcomes}}$$

For example, the probability of a coin landing heads is ½. There is one desired outcome (landing heads) out of two possible outcomes.

Question 1: Rolling Dice

Lucy rolls 2 fair six-sided dice, and adds the values of the two sides facing upward. What is the sum that she is most likely to receive?

Question 2: Arranging Books

Emma has 5 books that she wants to arrange on her bookshelf. She has 2 identical copies of “Introduction to Probability”, and 3 identical copies of “Everyday Geometry”. How many ways can Emma arrange the books on her shelf?

Question 3: Rolling Dice, Part 2

Lucy has two fair six-sided dice. Olivia also has a fair six-sided die. The girls roll all three dice. What is the probability that the sum of the face-up numbers on Lucy’s two dice equals the face-up number on Olivia’s die?

Question 4 (Challenge Question): Game Stand

You are operating a game stand. A contestant rolls two dice. If the sum of the two dice is greater than 9, they win 3 dollars. If the sum of the dice is less than 6, they lose “X” dollars. If the sum of their dice does not fit either requirement, they do not gain or lose any money.

Part A: Find the probability the sum of the two dice is greater than 9

Part B: Find the probability the sum of the two dice is less than 6.

Part C: Expected Value. Find “X”, (the amount of money lost if the sum of the dice is less than 6), such that the expected value of playing this game is -1 dollars (expected to lose 1 dollar).

> Hint: The “expected value” of our game stand is how much we would expect a contest to earn from playing the game once. Mathematically, the formula for expected value is:
>
> Expected Value = Value(Outcome 1) * Probability(Outcome 1) +
> Value(Outcome 2) * Probability(Outcome 2) +
> Value(Outcome 3) * Probability(Outcome 3) + ...
>
> In this case, you can use this equation to measure expected value:
>
> -1 = 3 * Probability(Sum > 9) + (- X) * Probability(Sum < 6)

Station 1: Probability (Volunteers)

Description

This is a station with general introductions to probability concepts. Participants will be asked to count, measure uncertainty, and (as a challenge) calculate expected value and payoffs.

Capacity	About 6-8 people at a time, try to keep within this range
Time	15 minutes
Completion (Basic) (1 stamp)	Finish problems 1 and 2 (or have shown significant effort/thought)
First Challenge (2nd stamp)	Finish problem 3 (must have correct solution, if participant has incorrect solution volunteer can provide guidance)
Completion (Challenge) (3rd and 4th stamp stamp)	Finish problem 4 (parts A and B = 1 stamp, part C = 2nd stamp) (must have correct solution, if participant has incorrect solution volunteer can provide guidance)

Instructions

1. Station Overview (3 minutes)
    a. Introduce probability (read introduction section on this sheet and on participant's sheet as well)
    b. Explain the completion criteria
2. Support Participants (12 minutes)
    a. Explain hints if needed
    b. People can write down process/calculations on paper and use cards so there is a visual/hands-on aspect
    c. For problem 2 and 3, encourage them to use props (tiles and dice) to try and discover the pattern
3. Hand out stamps
    a. Grade problems 1 to 3
    b. Not a timed station

Materials

- Answer key + hints (volunteers only)
- Paper with the questions (1 for each participant)
- Dice/cards/other interactive materials (to share)
- Blank paper for participants to use
- It's OK for participants to write answers on their worksheet

Probability Answer Key

Question 1: Rolling Dice
Lucy rolls 2 fair six-sided dice, and sums the values of the two sides facing up. What is the sum that she is most likely to receive?

Answer: **7**
Solution:
There are 11 possible sums Lucy can roll, with 2 being the smallest (rolling two ones) and 12 being the largest (rolling two sixes).
The probability of Lucy receiving either a 2 or a 12 is 1/36: 1 outcome for each (rolling two ones or sixes) among 36 possible outcomes (6 * 6 for each of the sides).
There are more possibilities for rolling sums between 2 and 12. We can outline each of the cases below:

Sum	Possibilities	Number of Possibilities
2	1,1	1
3	1,2 | 2,1	2
4	1,3 | 2,2 | 3,1	3
5	1,4 | 2,3 | 3,2 | 4,1	4
6	1,5 | 2,4 | 3,3 | 4,2 | 5,1	5
7	1,6 | 2,5 | 3,4 | 4,3 | 5,2 | 6,1	6
8	2,6 | 3,5 | 4,4 | 5,3 | 6,2	5
9	3,6 | 4,5 | 5,4 | 6,3	4
10	4,6 | 5,5 | 6,4	3
11	5,6 | 6,5	2
12	6,6	1

Looking at the table above, we see that there are 6 distinct ways to roll a sum of 7. Intuitively, this makes sense as the number halfway between 2 and 12 (or the *average*). With 6 possibilities, **7 is the most likely sum for Lucy to roll** (with a chance of 6/36 or ⅙).

Question 2: Arranging Books
Emma has 5 books that she wants to arrange on her bookshelf. She has 2 identical copies of “Introduction to Probability”, and 3 identical copies of “Everyday Geometry”. How many ways can Emma arrange the books on her shelf?

Answer: **10 Ways**
Solution 1: Counting by Hand
We can count the number of arrangements case-by-case. If Emma labels the two copies of “Introduction to Probability” with a **P** and the three copies of “Basic Geometry” with a **G**, the distinct ways she can arrange the books are:

1. PPGGG
2. PGPGG
3. PGGPG
4. PGGGP
5. GPPGG
6. GPGPG
7. GPGGP
8. GGPPG
9. GGPGP
10. GGGPP

The arrangements can be ordered according to this pattern:
- 1-4: Placing one P in spot 1, there are 4 spaces (2 through 5) to place the second P
- 5-7: Placing one P in spot 2, there are 3 spaces (3 through 5) to place the second P that avoids spot 1
- 8-9: Placing one P in spot 3, there are 2 spaces (4 and 5) to place the second P that avoids spots 1 and 2.
- 10: Placing one P in spot 4, there is 1 space (5) to place the second P that avoids spots 1, 2 and 3

Summing these cases together, we get **4 + 3 + 2 + 1 = 10**

Solution 2: Factorials
To solve this problem, we can also use the concept of a **factorial**. A “number factorial” is equivalent to that number multiplied by that number minus one, multiplied by that number minus two, and so on until 1. In mathematical notation, where n is the original number, “n factorial” (or “n!”) would be equal to n * n-1 * n-2 * … * 3 * 2 * 1.

Factorials have special connections to counting. There are “5!” ways to arrange 5 unique books in a line, which is 5 * 4 * 3 * 2 * 1 = 120. However, the books are not unique - there are 2 identical copies of “P” and 3 identical copies of “G”. This leads to “3! * 2!” duplicates for each arrangement. These duplicate arrangements can be divided from “5!” to arrive at the final answer: **5! / (3! * 2!) = (5 * 4 * 3 * 2 * 1) / (3 * 2 * 1 * 2 * 1) = (5 * 4) / 2 = 10**

Question 3: Rolling Dice, Part 2

Lucy has two fair six-sided dice. Olivia also has a fair six-sided die. The girls roll all three dice. What is the probability that the sum of the face-up numbers on Lucy's two dice equals the face-up number on Olivia's die?

__Answer:__ **__5/72__** (if participants arrive at **15/216**, encourage them to simplify)

Hint:
If the participants created a table of sums for question 1, they can use that table to help them solve this problem! It's also OK to help them compute the denominator 6^3 = 216.
Solution:
Similar to questions 1 and 2, Olivia can approach this problem by counting. Let's refer back to the table of sums from Question 1:

Sum	Possibilities	Number of Possibilities
2	1,1	1
3	1,2 | 2,1	2
4	1,3 | 2,2 | 3,1	3
5	1,4 | 2,3 | 3,2 | 4,1	4
6	1,5 | 2,4 | 3,3 | 4,2 | 5,1	5
7	1,6 | 2,5 | 3,4 | 4,3 | 5,2 | 6,1	6
8	2,6 | 3,5 | 4,4 | 5,3 | 6,2	5
9	3,6 | 4,5 | 5,4 | 6,3	4
10	4,6 | 5,5 | 6,4	3
11	5,6 | 6,5	2
12	6,6	1

The sum of the first two dice must equal the value of the third die. So the only applicable sums are sums 2 through 6. This leads to a total 5+4+3+2+1 desire outcomes, or 15. The number of possible outcomes rolling three dice is 6^3, which is 216. Thus, our answer is 15/216, which simplifies to **5/72.**

Question 4 (Challenge Question): Game Stand

You are operating a game stand. A contestant rolls two dice. If the sum of the two dice is greater than 9, they win 3 dollars. If the sum of the dice is less than 6, they lose “X” dollars. If the sum of their dice does not fit either requirement, they do not gain or lose any money.

Part A: Find the probability the sum of the two dice is greater than 9

Answer: **1/6** (or 6/36)

Solution:

Using the table from Question 1, there are 3 ways to roll a sum of 10, 2 ways to make 11 and 1 way to make 12. This sums up to 6 desired outcomes, for a probability of 6/36 or 1/6.

Part B: Find the probability the sum of the two dice is less than 6.

Answer: ***5/18*** *(or 10/36)*

Solution:

There are 4 ways to roll a sum of 5, 3 ways to make 4, 2 ways to make 3 and 1 way to make 2. This sums up to 10 desired outcomes, for a probability of 10/36 or 5/18.

Part C: Expected Value. Find “X”, (the amount of money lost if the sum of the dice is less than 6), such that the expected value of playing this game for a contestant is -1 dollars.

> Hint: The “expected value” of our game stand is how much we would expect a contest to earn from playing the game once. Mathematically, the formula for expected value is:
> Expected Value = Value(Outcome 1) * Probability(Outcome 1) +
> Value(Outcome 2) * Probability(Outcome 2) +
> Value(Outcome 3) * Probability(Outcome 3) + ...
> In this case, you can use this equation to measure expected value:
>
> -1 = 3 * Probability(Sum > 9) + (- X) * Probability(Sum < 6)

Answer: **X = $5.40**

Solution:

We’ve calculated the two probabilities in Part A and Part B. Plugging those values in, we get:

-1 = 3* (1/6) - X *(5/18)

Solving for X, we get:

-1 = 1/2 - X * (5/18)

-3/2 = - X *(5/18)

3/2 = X * (5/18)

3/2 * (18/5) = X

Solving for X gives 27/5, or **$5.40.**

Station 2: Chess (Participants)

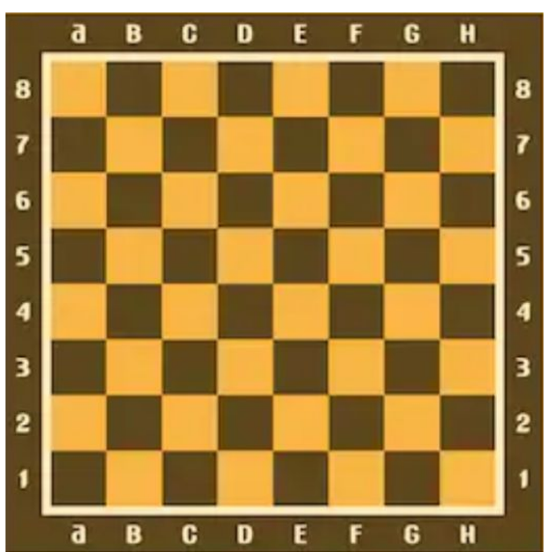
Authors: Laasya Nagareddy, Shifa Somji, Luke Xie

Question 1: Move #2

Suppose you are playing for the white side and begin a chess match by moving your knight. How many possible moves can you take for your second move if you don't move the same knight once more?

Question 2: Queens

What is the minimum number of queen chess pieces that you can put on a blank chess board so that at least one queen can move to any square on the board in the next turn?



Station 2: Chess (Volunteers)

This is a station in which students will use interactive materials to determine various properties of chess and understand how pieces move.

Capacity	About 6-8 people at a time, but no maximum
Estimated Time	15 minutes
Completion (1st stamp)	Finish problem 1 with the correct solution Partially finish problem 2 (participants should have displayed significant effort/thought)
Challenge (2nd Stamp)	Finish Problem 2 with the correct answer and a correct example of the solution

Instructions

1. Station Overview (3 minutes)
    a. Introduce chess (reading the necessary moves as follows):
        i. A pawn, on its first move, can move one or two spaces forward
        ii. A bishop can move along unobstructed diagonals
        iii. A rook can move along unobstructed rows or columns
        iv. A queen can move along unobstructed rows, columns or diagonals
        v. A knight can jump in a 2x1 "L" shape (show by example)
    b. Explain the completion criteria (can earn up to 2 stamps) (see above)
    c. Briefly remind students of the rules of chess
    d. Students will work to solve the 2 problems on the worksheet, volunteers can encourage collaboration and discussion
    e. Participants can use the paper chessboard or one of the real ones for reference.
2. Support Participants (12 minutes)
    a. Explain hints if needed
    b. People can write down process/calculations on paper
    c. For problem 2, encourage them to color squares on the blank chessboard.
    d. Can offer help to kids who might need hints
3. Hand out stamps
    a. Grade problems 1 and 2
    b. Not a timed station

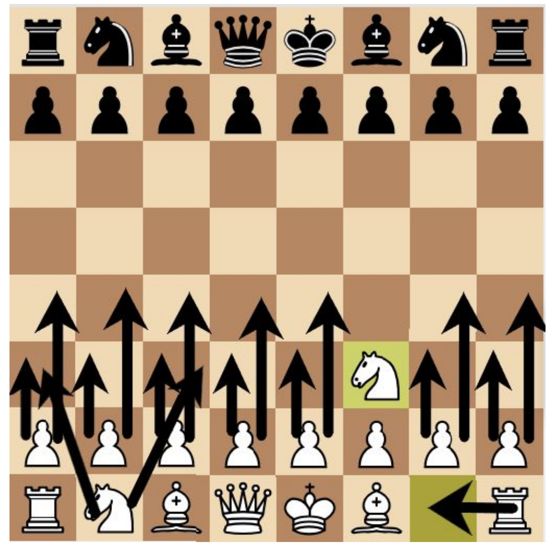
Materials

- Answer key + hints (volunteers only)
- Paper with the questions and scratch paper (1 for each participant)
- Interactive materials (chessboard) (to share)

Chess Answer Key

Question 1: Move #2

Answer: **17 moves**



- Each pawn, except for the one blocked by the knight, has 2 possible moves. There are 7 pawns not blocked by the knight.
- The rook that was next to the knight that was moved can move over one square.
- The other knight that was not moved has 2 possible moves

Total: 2 x 7 (pawns) + 1 (rook) + 2 (other knight) = 17 possible moves

Question 2: Queens

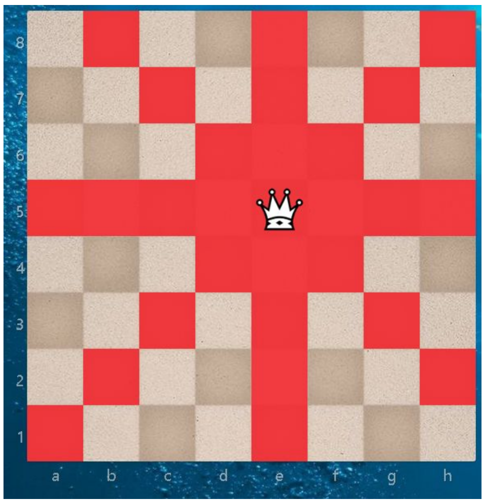
Answer: **five queens**

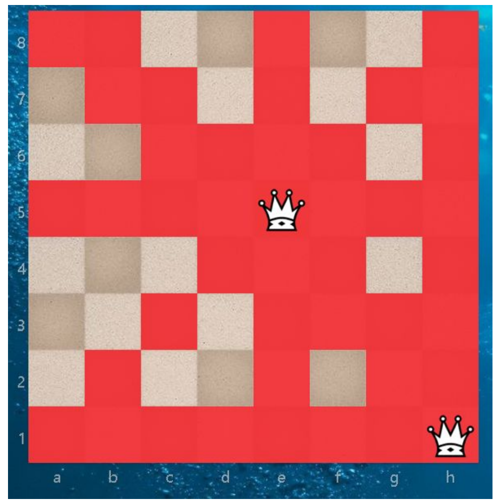
One example of a possible solution:

						Q	
Q							
				Q			
			Q				
							Q

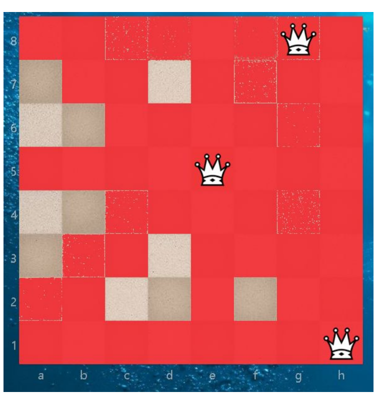
Solution:
At the very most 8 queens are needed to cover each row, and thus cover all 64 squares.
But since queens move in diagonals, up and down, and side to side, fewer queens are needed to cover the entire board. There are various ways you can solve this. The general strategy is to start with a queen in the center to control as many squares as possible. The most important thing is to take advantage of the multidirectional control of the queen.

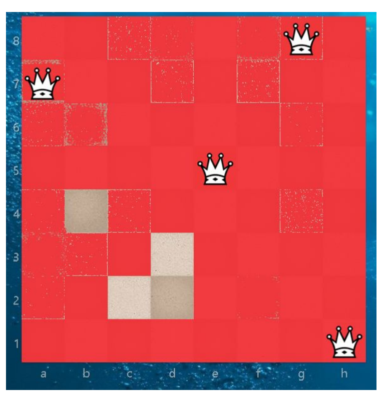
The queen in the center of the left board below controls 28 out of the 64 squares on the board!

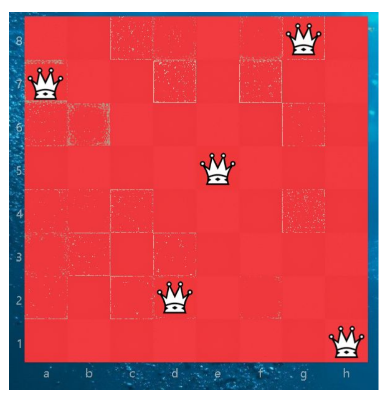




From there, we want to add another queen to continue covering as many squares as possible. Going to one of the corners, we can control 15 more squares, leaving 21 squares yet to be controlled (on the right board above). We take the strategy of maximizing control along uncovered diagonals, rows and columns to arrive at the final solution.

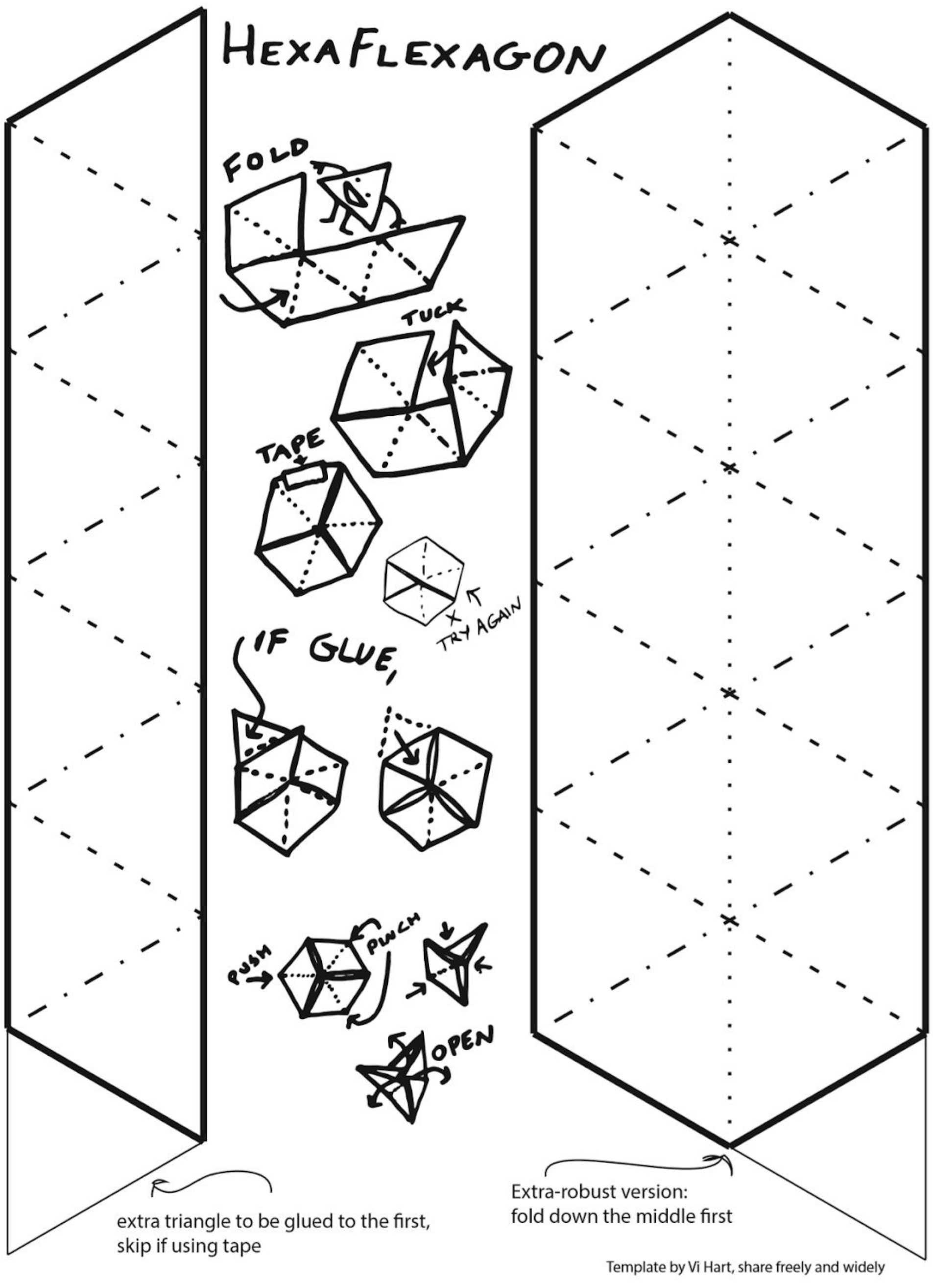






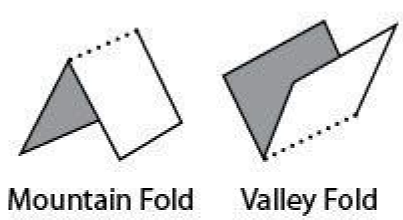
Station 3: Origami (Participants)

Authors: Vivian Lu, Laasya Nagareddy

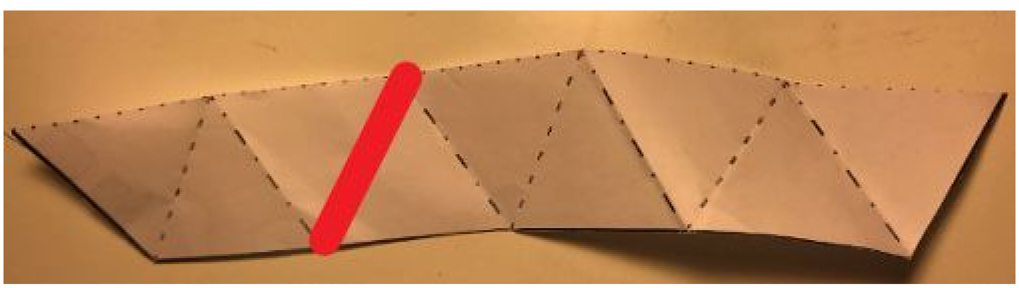


Hexaflexagon Folding Instructions

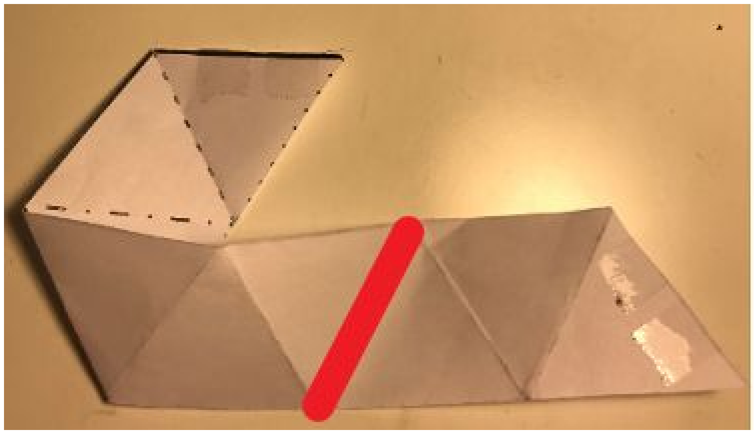
1. Cut out the template on the **left side** of the sheet, cutting around the bold edges. Do not cut out the extra triangle, we will be using tape instead of glue. If you mess up on the first try, there are more templates on the right side of the sheet.
2. Notice the dashed lines on the templates that create individual equilateral triangles. One of the dashed lines looks like this: - - - - - - - - - - - , which we will call type 1. The other dashed line looks like this: — * — * — *—, which we will call line type 2.
3. Fold mountain folds along line type 1 (- - - - - - - - -), and valley folds along line type 2 (— * — * — * —). ***Make sure on each line to only fold in one direction.**



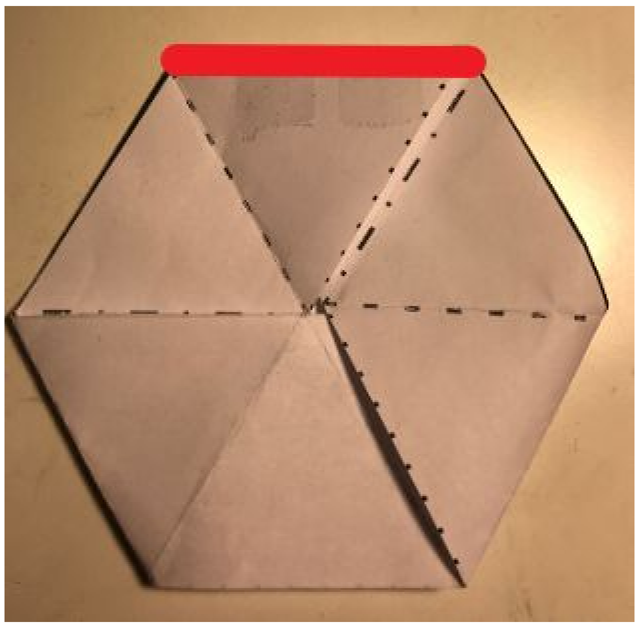
4. Next, your paper should like this. Fold along the red line so your paper looks like the image in Step 5.



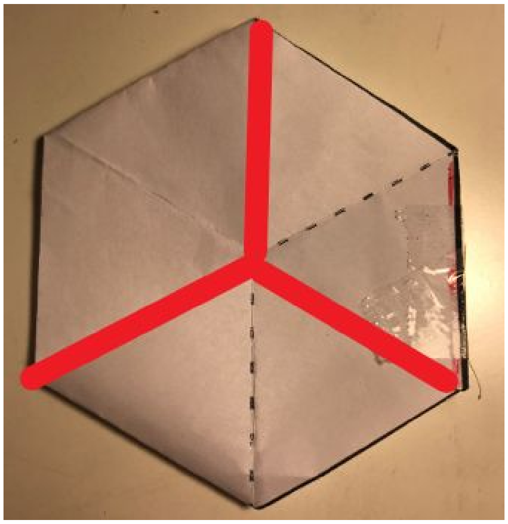
5. After Step 4, your paper should look like this. Fold along the red line again and this time tuck the end of the paper strip behind the first triangle so it looks like the image in Step 6.

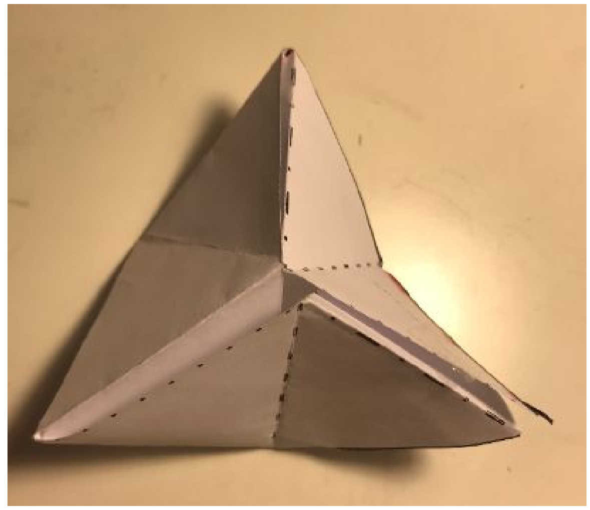


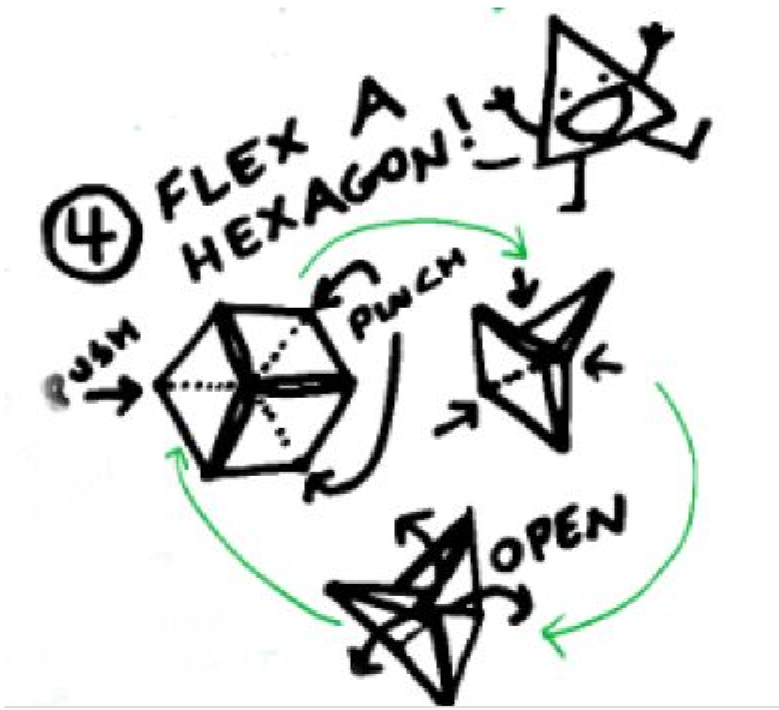
6. Your paper should now look like the image below. Put tape along the edge indicated by the red line to attach the ends of the paper together.



7. Congratulations! You have completed folding the hexaflexagon.
8. Now let's explore some properties of the hexaflexagon. Fold inwards on the three red lines to get a triangular shape as we see in the image on the right.







9. Once we get to the triangle figure (as seen above), we should notice that the center opens up again! In fact, we can repeat step 8 over and over and the same thing happens.

Station 3: Hexaflexagons/Origami (Volunteers)

Description

This station will have participants folding a hexaflexagon from a given template and exploring its properties.

Capacity	About 6-8 people at a time, try to maintain within this range
Estimated Time	15 minutes
Completion (Basic) (1 stamp)	Finish folding hexaflexagon and participants either have a written or verbal answer for Question #3 on the worksheet

Instructions

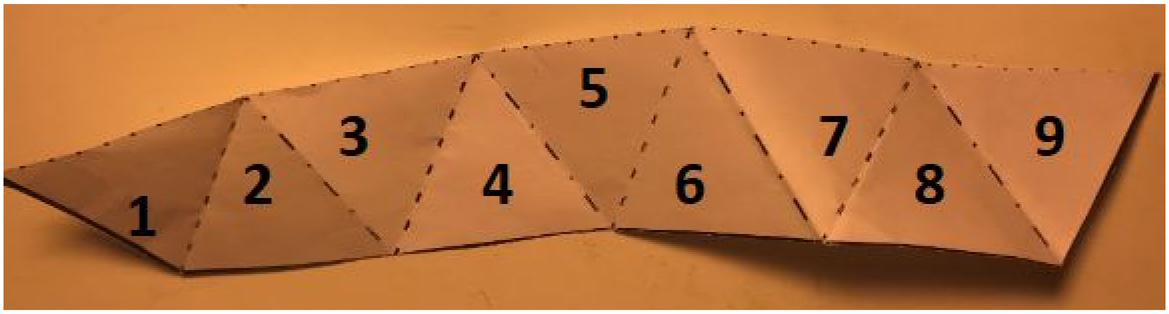
1. Folding the Hexaflexagon
    a. Give each participant a template for the hexaflexagon and an instruction packet for folding (packets must stay at the station)
    b. Make sure you understand how to fold the hexaflexagon as well, so you can advise participants if they get confused
    c. Participants should know they will get a worksheet once they show they have completed folding a hexaflexagon
2. Worksheet/Discussion
    a. Each participant should get a copy of the worksheet: “Exploring the Hexaflexagon’s properties” after finishing folding their hexaflexagon
    b. Let each participant go through the exercises on: “Exploring the Hexaflexagon’s properties”
    c. Participants should know they should either have a verbal or written explanation for question #3. They do not necessarily need to reach a “solution” (such as the one on the answer sheet), however they should indicate some effort/thought was paid.
    d. There is an answer sheet for you to explain the answer after the participant thinks they have the answer or if the participant is unsure. You can simply read off the answer sheet. You can also show the participants the images on the answer key to aid their understanding.

Materials

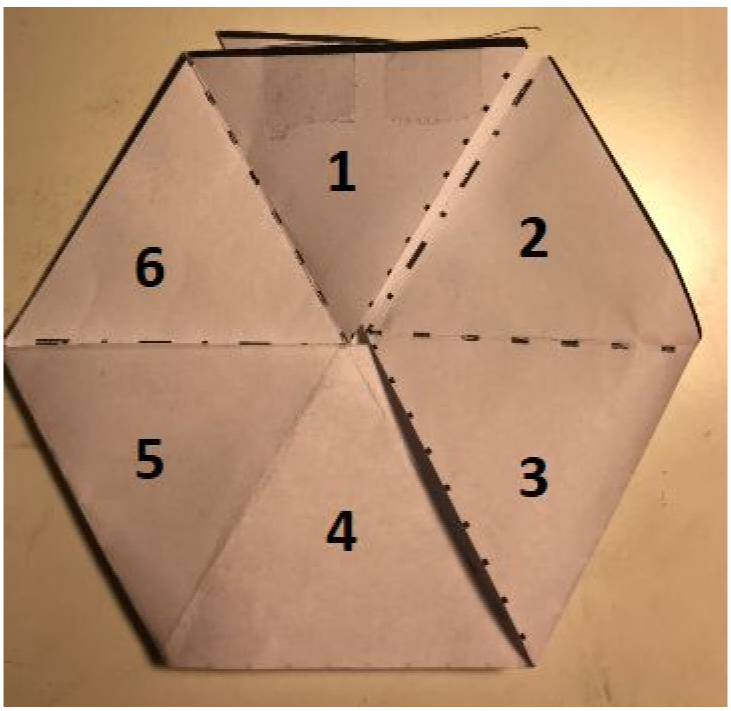
- Station Instructions + Answer Key (For volunteers only)
- Hexaflexagon Instruction Packets
- Hexaflexagon Template Papers
- Hexaflexagon Worksheets
- Tape, Markers, Scissors

Answers: Exploring the Hexaflexagon's Properties

- If you color each blank side of the hexaflexagon (by coloring in each blank hexagon you see), you should have ended up using 3 colors.
- Basically, this means that there are 3 sides of the hexaflexagon, which was initially created from a two-sided paper.
- This can be explained by going back to our original template. When looking at one side of the paper, there are 9 equilateral triangles.



- When we flip the paper over, the other side also has 9 equilateral triangles.
- Therefore we can consider that the paper is composed of 18 faces of equilateral triangles, or 9 equilateral triangles with two faces.
- When looking at our hexaflexagon, each hexagon face has 6 triangles



- Since there are 3 sides/faces to the hexaflexagon, and each face of the hexaflexagon has 6 equilateral triangles, the hexaflexagon also has 3 * 6 = 18 faces of equilateral triangles or 3 faces with 6 equilateral triangles each.

Station 4: Sudoku (Participants)

Author: Christina Yao

Rules

The classic Sudoku game involves a grid of 81 squares, divided into 9 blocks of 9 squares.

- Each of the nine blocks has to contain all the numbers 1-9 within its squares. Each number can only appear once in a row, column or box.
- Each vertical nine-square column, or horizontal nine-square line across, within the larger square, must also contain the numbers 1-9, without repetition or omission.
- Every puzzle has just one correct solution

1		6	7			4		9
9		7			3		2	
	2			4	9			
			2			3	6	
6								4
	1	9			6			
			4	3			5	
	6		9			2		3
3		4			1	9		8

Extension Puzzle

		6	4	8	1	3		
	2						4	
7								9
8				9				4
6			3	4	2			1
5				6				2
3								5
	9						7	
		5	7	1	6	2		

Station 4: Sudoku Volunteer Instructions

Description

This station will have participants solving sudoku puzzles. They will try to solve it as fast as possible, testing their speed and accuracy.

Capacity	About 6-8 people at a time, but no maximum
Estimated Time	15 minutes
Completion (1st stamp)	Finish problem 1 with the correct solution
Challenge (2nd Stamp)	Finish Problem 2 with the correct answer

Instructions

- This station is a competitive station, which means the round begins every 20 min.

  General Schedule (may be subject to change)

1:30-1:50	First round
1:50-2:10	Second
2:10-2:30	Third
2:30-2:50	Fourth
2:50-3:10	Fifth
3:10-3:30	Sixth
3:30-4:00	Rewards
- Materials: pencils, waiting-list for students to sign-up for the next round (they can also choose to drop-by at the beginning of the round if the tables are not full), timer
- This station will have 2 tables; each table can only have at most 5 students
- Briefly explain to students of the rules of Sudoku (explanation on the worksheet) and explain the rule for stamps for this station (see below)
- Students are going to work on either Sudoku puzzle within the 15 minutes (the remaining 5 minutes are for clean-up and getting ready for the next round)
- Record everyone's time with names and which sudoku puzzle they worked on a spreadsheet (ask if they want to enter the competitive round)--this is for the top 3 fastest out of all participants for each puzzle.
- Students can only try each puzzle once in the next few rounds for stamps
- Each student can earn 2 stamps maximum, one for each problem.
- Volunteers can offer help to kids that are struggling
- Students cannot leave the table with the Sudoku puzzle

Original Puzzle Answer Key

1	8	6	7	5	2	4	3	9
9	4	7	1	6	3	8	2	5
5	2	3	8	4	9	6	1	7
8	7	5	2	9	4	3	6	1
6	3	2	5	1	8	7	9	4
4	1	9	3	7	6	5	8	2
2	9	8	4	3	7	1	5	6
7	6	1	9	8	5	2	4	3
3	5	4	6	2	1	9	7	8

Extension Puzzle Answer Key

9	5	6	4	8	1	3	2	7
1	2	3	6	7	9	5	4	8
7	4	8	2	5	3	6	1	9
8	3	2	1	9	5	7	6	4
6	7	9	3	4	2	8	5	1
5	1	4	8	6	7	9	3	2
3	6	7	9	2	4	1	8	5
2	9	1	5	3	8	4	7	6
4	8	5	7	1	6	2	9	3

Station 5: Tangrams (Volunteers)

Authors: Vivian Lu, Christina Yao, Anish Nagareddy

NOTE: At the event, used the following set of tangram pieces:
https://www.amazon.com/gp/product/B00000K3BU/ref=ppx_yo_dt_b_asin_title_o02__o00_s00

Description

This station will have participants making tangrams. The tangram is a dissection puzzle consisting of seven flat shapes, called tans, which are put together to form shapes. The objective of the puzzle is to form a specific shape using all seven pieces, which cannot overlap.

Capacity	About 6-8 people at a time, depending on number of tangram sets available
Estimated Time	15 minutes
Completion (Basic) (1 stamp)	Finish 3 tangrams (drawn on worksheet)

Instructions

1. Each participant has their own set of tangrams
    a. Make sure that each set has seven geometric shapes
    b. There are also cards with different shapes that the participants are trying to create
    c. Tell participants: the goal is to make the shapes on the cards using the seven geometric shapes in the tangram set
    d. Tell participants: choose a card and try to make the shape on that card using the shapes in your set
2. Requirements for participants to get stamps
    a. Participants must solve 3 of the tangram puzzles (on the cards) and have drawn them out on their worksheet. Volunteers should confirm this has been done before giving out a stamp.
    b. Make sure participants are aware of the challenge: finish 3 more tangrams to get a second stamp

- Note: Make sure the pieces for each tangram puzzle is kept separate to each puzzle,

Materials

- Tangram Volunteer Instructions (1 per volunteer)
- Tangram sets: 7 boxes, deck of cards, 4 sets of 7 shapes
- Tangram Worksheets (1 for each participant)

Station 6: Twenty-Four (Volunteer Instructions)

Authors: Laasya Nagareddy, Vivian Lu

Students will use mathematical operations to get the number 24 from the numbers drawn from 4 cards. This is a competitive station (in rounds of 20 minutes) and competitors are awarded based on merit (points scored).

Capacity	About 6-8 people at a time per round of the game
Estimated Time	20 minutes competitive rounds, followed by more rounds (follow the schedule for round timings)
Completion (Basic)	A participant gets 1 point for being the first to provide a solution for the given problem. At the end of the 20 minute round, the person with the most points gets **2 stamps**, and the person with the second most points gets **1 stamp**. (In the case of a tie for first, give both participants 2 stamps, in the case of a tie for second, give both participants 1 stamp)

Instructions

1. Explain Rules
    a. In each round, the volunteer will draw a card from the deck of cards provided and read off the 4 numbers on the card.
    b. Using addition, subtraction, multiplication, division, and exponents, your goal is to create an equation with the four numbers given that equals the number 24.
    c. Once a participant has an answer, they should write it on their whiteboard, raise their hand, and have the volunteer verify the solution (requires a bit of mental math for volunteers). A participant receives a point for being the first person to get a correct answer. If the first participant to answer is incorrect, allow other participants a chance to get the correct answer.
    d. Each participant gets their own whiteboard and pen, the whiteboards can be used to write calculations and present the final answer
    e. Note: Allocate 1-2 minutes for participants to think about their answer
        i. If there are no answers after the time, volunteers can either provide a hint if they know the solution or move on to another question
2. Explain requirements for stamps (seen above)
    a. For volunteers: make sure you are keeping track of how many points each person has, such as by writing the point totals on a blank piece of paper

Materials:

- 24 game card deck
- Mini Whiteboards and Dry Erase Markers

Station 7: Bingo (Volunteers)

Authors: Laasya Nagareddy, Anish Nagareddy, Vivian Lu

This station will have participants solving bingo puzzles. Volunteers will be reading out questions and using mental math to get a bingo.

Capacity	About 6-8 people at a time, but no maximum
Estimated Time	15 minutes
Completion (1st stamp)	A participant gets two bingos (four squares in a row) (volunteer should confirm that the numbers on the participant's bingo have been answers to the questions)

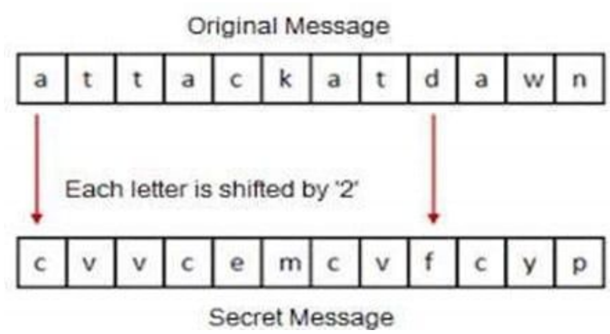
Instructions for volunteers

- This station is a competitive station, students will be given a bingo board and solve equations using mental math
- Volunteers will provide students with a bingo board, and a pencil
- Volunteers will read out the basic instructions and give each student a bingo card.
    - Volunteers will read out an equation/question (in random order), and students will solve the problem and cross off the respective bingo square.
    - Wait between 10-20 seconds after reading one question and then move on to another question
    - Students will fill in the square of the solution to the equation (students should try to solve the equations with only mental math
    - (if a student has multiple of the same number on their card, each time one of those numbers is called only counts for filling in 1 square) (ex: if the answer to the question is "12" and the student has 2 squares that have the number 12, they can only fill in 1 square)
    - 2 Bingos = 1 stamp
    - **NOTE:** if you draw a question that has the same answer as the previous question, choose another question instead of reading that question
- Keep track of the answers you read off (suggestion: write the answers to the questions you have read on a blank sheet of paper)
    - Each answer you read should count for 1 square (make sure to keep track of repeats!!!)

Station 8: Codebusters (Participants)

Authors: Laasya Nagareddy, Simon Kwon

The Caesar cipher is one of the earliest known and simplest ciphers. It is a type of substitution cipher in which each letter in the plaintext is 'shifted' a certain number of places down the alphabet. For example, with a shift of 1, A would be replaced by B, B would become C, and so on. The method is named after Julius Caesar, who apparently used it to communicate with his generals.



The Affine cipher is an alphabetic substitution cipher. The letters a - z are assigned to number values:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

The numbers are then inputted into a function with a linear equation. The outputs are divided by 26, and the remainder, which should be a number from 0 - 25, is the value for the coded letter. The remainder can also be expressed by "mod 26" (seen below). An example using the equation 9x + 42 is shown below:

Affine Cipher Encryption

Plaintext	C	O	D	E	B	U	S	T	E	R	S
x	2	14	3	4	1	20	18	19	4	17	18
$9x + 42$	60	168	69	78	51	222	204	213	78	195	204
$(9x + 42) \bmod 26$	8	12	17	0	25	14	22	5	0	13	22
Ciphertext	I	M	R	A	Z	O	W	F	A	N	W

Problems:

1. Decode this text using Caesar Cipher with a shift of 3: **PDWK LV DZHVRPH**

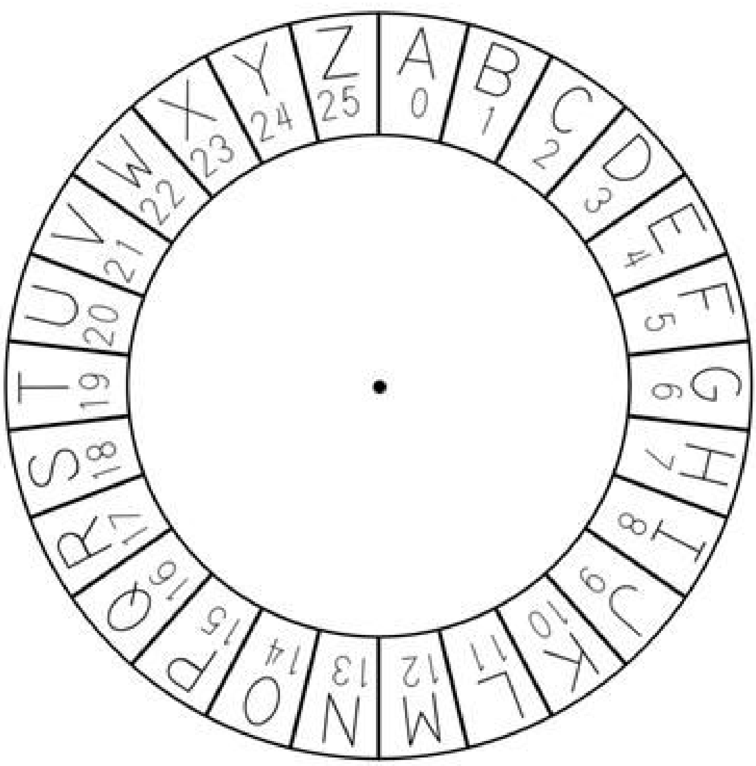
2. When using an Affine Cipher Encryption, what is the linear equation used to get from the original sentence to the encrypted sentence?)

   Original Sentence: **THE STRANGER OFFICIATES THE MEAL**

   Encrypted Sentence: **PFW MPJKXCWJ AXXIQIKPWM PFW UWKR**

3. **(Challenge)** Use the equation you found from question #2 to decode this word:

   QIDFWJM KJW VWJE IXPWJWMPIXC



Station 8: Codebusters (Volunteers)

This is a station with general introductions to ciphers and codes. Participants will learn about two ciphers, the Caesar Cipher, and the Affine Cipher, and use these to encode and decode problems.

Capacity	6-8 people
Time	20 minutes
Completion (Basic) (First Stamp)	Finish problems 1 and 2
Completion (Challenge) (Second Stamp)	Finish the challenge (part 2 of problem 2)

Instructions

1. Station Overview (5 minutes)
   a. Introduce ciphers (description at top of problem sheet)
   b. A quick overview of the modulus (dividing and finding remainder)
   c. Explain completion criteria (1 stamp for the correct solution to both problems, another stamp for correction solution to challenge)
   d. Hand out problem sheets and cipher wheels (1 for each student)
2. Support Participants (15 minutes)
   a. Explain hints if needed
   b. Students can use cipher wheels and scratch paper
   c. Encourage writing out process, especially for Affine Cipher problems
3. Hand out stamps
   a. Grade problems 1, 2 and challenge

Materials

- Description sheet and solution sheet (volunteers only)
- Problem sheets (1 for each participant)
- Cipher wheels cut out (1 set for each participant)
- Blank paper, pencils
- Participants can write answers on the worksheet

Codebusters Answer Key

Decode using Caesar Cipher with a shift of 3: PDWK LV DZHVRPH

- Caesar cipher of shift 3: “Math is awesome”
- Hints: They should use the table given

Decode the linear equation used for this Affine Cipher Encryption.
Original Sentence: The stranger officiates the meal.
Encrypted Sentence: PFW MPJKXCWJ AXXIQIKPWM PFW UWKR

- Affine Cipher Solution with translation equation 3x + 10 (mod 26): PFW MPJKXCWJ AZZIQIKPWM PFW UWKR
- Hints: The letter “A” in the original sentence is a special letter. What property does it have? How do you determine the equation of a linear line?

Challenge: use the same code to decode QIDFWJM KJW VWJE IXPWJWMPIXC

- Using the equation 3x + 10 (mod 26): “Ciphers are very interesting”
- Hints: Follow the procedure of assigning numbers to letters, inputting them into the equation, dividing by 26 and finding remainder.

Station 9: Triangles (Volunteers)

Authors: Christina Yao, Laasya Nagareddy, Vivian Lu

Students will be given a paper triangle and will use various mathematical processes to turn the triangle into a square.

Capacity	About 6-8 people at a time, but no maximum
Estimated Time	15 minutes
Completion (1st stamp)	Make significant progress or finish the problem completely

Instructions

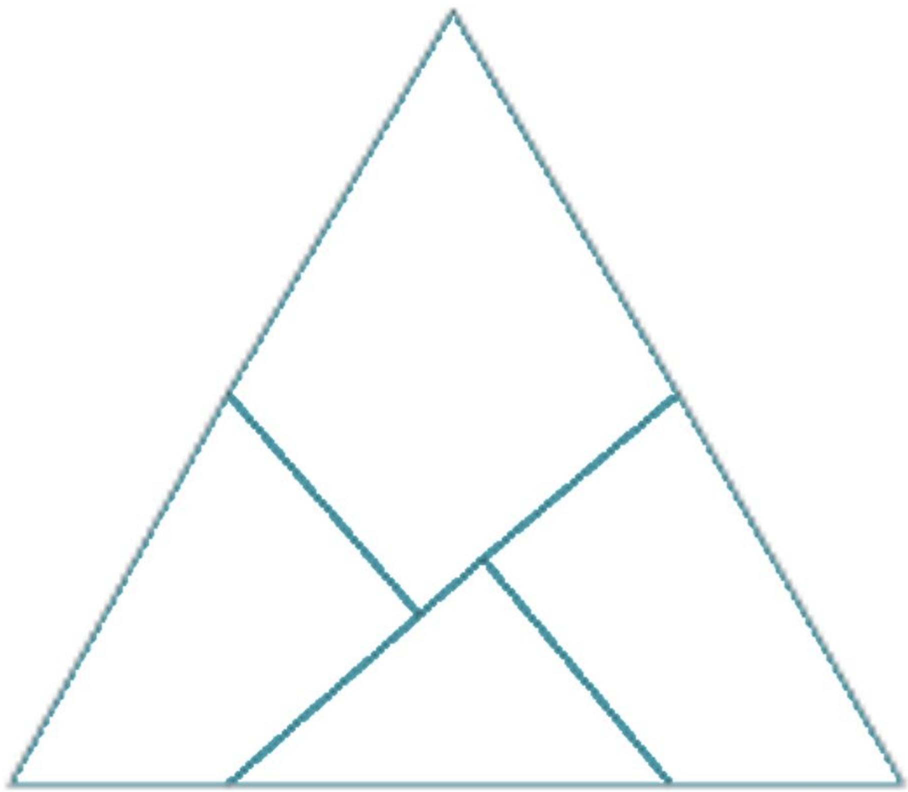
- This station is not a competitive station
- Students will be given a set of four pieces that can be arranged into both a triangle and a square.
- Volunteers will provide:
    - Various cutouts of shapes
    - Pages with a square for students to place the cutouts on
    - Pencils or paper to work with, will probably not be needed
    - A waiting list for students to sign up ~ drop-ins should be fine
- Student will be trying to use the pieces of paper given to form a square
    - They will be given multiple cut-out pieces and be told to form a square
    - The solution entails the use of four cut out pieces but many extras will be provided to increase complexity and promote problem-solving
    - After solving, they will show their solutions to the volunteers for stamps
- This station will have 2 tables; each table will have up to 5 students
- Explain the rules of the challenge: the students must arrange the given pieces into a triangle, and a square

Materials

- Pencils
- Pieces of puzzle
- Blank paper for participants to use
- Pages with a square for students to place cutouts on
- Waiting list for students

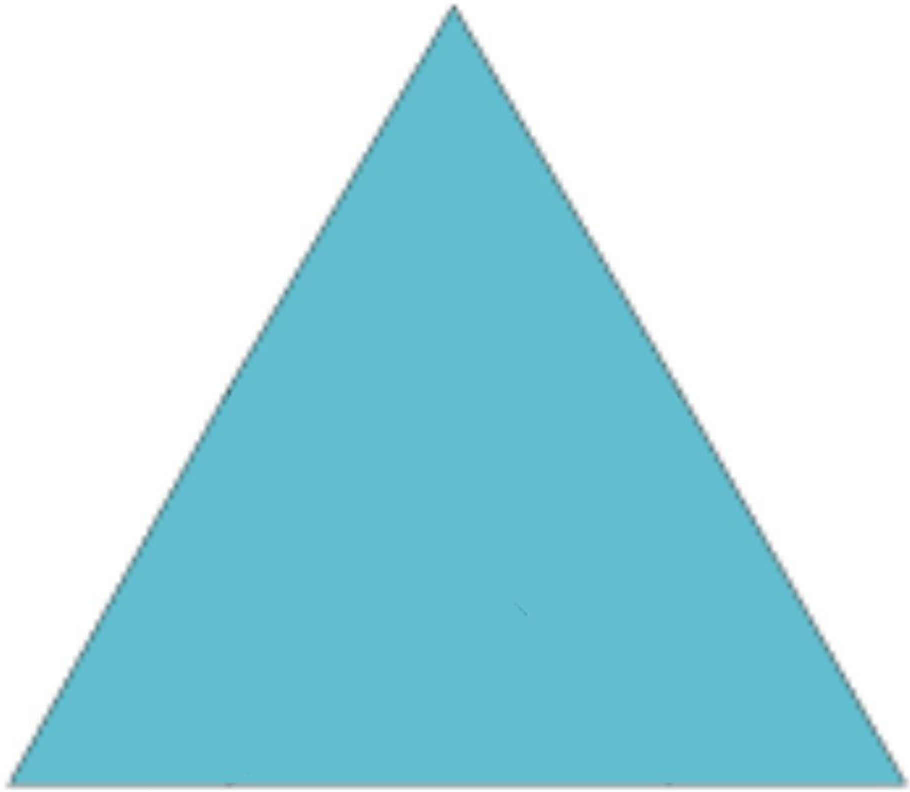
Worksheet: Triangles

Instructions (parent or instructor): Cut the triangle below into four pieces as outlined



Worksheet: Triangles

Instructions (Student): Given the pieces from above, try to arrange them in a non-overlapping fashion so that they fit the square and equilateral triangle in the following templates



Station 10: Dimensional Analysis (Participants)

Author: Pravir Chugh

1. How many cups of water are in Lake Erie?
   (if Lake Erie has a volume of **115.2 cubic miles)**

2. How long in seconds would it take to count to one billion?
   (If we assume it takes on average **2 seconds** to count 1 number)

3. How many beats will your heart make in 10 years?
   (If we assume the average heart beats at **75 beats per minute**)

Dimensional Analysis Extension

4. How many paper clips would it take to weigh the same amount as an elephant?
   (If we assume a paper clip is **1 gram**, and a elephant weighs **5 tons**)
   Hint: **1 kilogram = 2.2 pounds**

5. How many seconds are in the average human's lifetime?
   (If we assume the average human lives for **79 years**)

Conversion Factors

1 cubic mile = 1.101 * 10^12 gallons or 1101000000000 gallons	1 kilogram = 1000 grams
1 pint = 2 cups	1 kilogram = 2.2 pounds
1 quart = 2 pints	1 ton = 2000 pounds
1 gallon = 4 quarts	1 year = 365 days
1 minute = 60 seconds	1 day = 24 hours
1 hour = 60 minutes	1 pound = 0.453592 kilograms

Station 10: Dimensional Analysis (Volunteers)

Description

This station includes applications of estimation and dimensional analysis that can be applied to the real world. These are problems that require knowledge of certain conversion factors and interesting facts.

Capacity	About 6-8 people at a time, but no maximum
Time	15-20 minutes
Completion (Basic) (1 stamp)	Worksheet Progress, one stamp for completion or effort on problems 1-3
Completion (Advanced) (2 stamps)	Worksheet Progress, one extra stamp for completion or effort on all the problems (work on all problems)
Other Facts	• People may use paper/pencil and a calculator (if needed). • Give hints (conversion factors - if needed)

Instructions

1. Station Overview (3 minutes)
    a. Introduce dimensional analysis (read introduction section on participant sheet)
    b. Explain the completion criteria (both basic and advanced)
2. Support Participants (20-30 minutes)
    a. Explain conversion factors if needed. Participants should not need to know what these concepts mean, just how to handle conversions, etc.
    b. Participants should be aware that they can refer to “Conversion Factors” sheet
3. Hand out stamps
    a. First stamp: reasonable effort or completion for problems 1-3
    b. Second stamp: reasonable effort or completion for problems 4-5

Materials

- 3 copies of Volunteer Instructions, 3 Answer Keys
- Dimensional Analysis Worksheet (1 for each participant)
- Blank paper for participants to as scratch paper
- It’s OK for participants to write answers on their worksheet
    - In fact, encourage students to present their answers in an organized manner
    - Can determine reasonable effort based on work shown
    - If a student only has the numerical answer and no work, the answer must be correct

Introduction to Dimensional Analysis

Dimensional Analysis is a problem-solving method that uses the fact that any number or expression can be multiplied by one without changing its value. It is used to get from one unit of measurement to another.

For example, if we were given the question **"How many seconds are in a day?"**, we can break this down by steps:

1. Ask yourself, **"What units of measure do I want to know or have in the answer?"** In this problem you want to know "seconds in a day." Translating this to math, we want to know:

$$\frac{\text{seconds}}{\text{day}} \quad \text{or} \quad \frac{\text{sec}}{\text{day}}$$

2. Ask, **"What do I know?"** What do you know about how "seconds" or "days" relate to other units of time measure?

We know:

- 1 minute = 60 seconds
- 1 hour = 60 minutes
- 1 day = 24 hours

Therefore, the values below all equal 1:

$$\frac{60\text{ sec}}{1\text{ min}} \quad \frac{1\text{ min}}{60\text{ sec}} \qquad \frac{60\text{ min}}{1\text{ hr}} \quad \frac{1\text{ hr}}{60\text{ min}} \qquad \frac{1\text{ day}}{24\text{ hr}} \quad \frac{24\text{ hr}}{1\text{ day}}$$

3. Therefore, we can write the conversion from 1 day to x seconds (the number of seconds in a day) as follows:

$$1\,day * \frac{24\,hrs}{1\,day} * \frac{60\,minutes}{1\,hr} * \frac{60\,seconds}{1\,min} = 86400\,seconds$$

Notice that we start with our initial value of 1 day and multiply by conversion factors that are all equivalent to 1 to get our final value of 86400 seconds.

All dimensional analysis problems follow this same process but vary in the conversion factors that are used.

Station 10: Dimensional Analysis Answers

How many cups of water are in Lake Erie?
Answer: 2.029 * 10^15 cups of water

What we know:

- Lake Eries is 115.2 cubic miles in volume
- 1 cubic mile = 1.101 * 10^12 gallons or 1101000000000 gallons
- 1 pint = 2 cups
- 1 quart = 2 pints
- 1 gallon = 4 quarts

$$115.2\ cubic\ miles * \frac{1.101 * 10^{12}\ gallons}{1\ cubic\ miles} * \frac{4\ quarts}{1\ gallon} * \frac{2\ pints}{1\ quart} * \frac{2\ cups}{1\ pint} = 2.029 * 10^{15}\ cups$$

How many years would it take to count to one billion?
Answer: 63.41 years

- We assume that it takes on average 2 seconds to count one number
- 1 minute = 60 seconds
- 1 hour = 60 minutes
- 1 day = 24 hours
- 1 year = 365 days

$$1\ billion\ counts * \frac{2\ seconds}{1\ count} * \frac{1\ minute}{60\ seconds} * \frac{1\ hour}{60\ minutes} * \frac{1\ day}{24\ hours} * \frac{1\ year}{365\ days} = 63.41\ years$$

How many beats will your heart make in 10 years?
Answer: 394,200,000 beats in 10 years

- The average heart beats at 75 beats per minute
- 1 year = 365 days
- 1 day = 24 hours
- 1 hour = 60 minutes

$$10\ years * \frac{365\ days}{1\ year} * \frac{24\ hours}{1\ day} * \frac{60\ minutes}{1\ hour} * \frac{75\ beats}{1\ minute} = 394{,}200{,}000\ beats$$

How many paper clips would it take to weigh the same amount as an elephant?
Answer: 4,545,454.5 paper clips

To solve this problem, we need to know:

- The weight of a paper clip (about 1 gram)
- The weight of an elephant (about 5 tons)
- 1 ton = 2000 pounds
- 1 kilogram = 2.2 pounds
- 1 kilogram = 1000 grams

$$5\ tons * \frac{2000\ pounds}{1\ ton} * \frac{1\ kilogram}{2.2\ pound} * \frac{1000\ grams}{1\ kilogram} * \frac{1\ paperclip}{1\ gram} = 454{,}5454.5\ paper\ clips$$

How many seconds are in the average human's lifetime?
Average: 2,491,344,000 seconds

- Average human lives for 79 years
- 1 year = 365 days
- 1 day = 24 hours
- 1 hour = 60 minutes
- 1 minute = 60 seconds

$$79\ years * \frac{365\ days}{1\ year} * \frac{24\ hours}{1\ day} * \frac{60\ minutes}{1\ hour} * \frac{60\ seconds}{1\ minute} = 2{,}491{,}344{,}000\ seconds$$